Global U(1) Invariance and Mass Scales in Supersymmetry

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Abstract

In a supersymmetric theory with global U(1) invariance, the spontaneous breaking of the latter without the breaking of supersymmetry is revisited in the case of the most general superpotential of 3 singlet superfields. The interesting possibility of having 2 hierarchical mass scales is pointed out, together with its consequences as applied to the axionic solution of the strong CP problem.

1 Introduction

The spontaneous breaking of a continuous global symmetry is well-known [1] to result in a massless Nambu-Goldstone boson. If this happens in a supersymmetric theory [2] without the breaking of the supersymmetry, a massless superfield must emerge, resulting in the existence of "flat directions" [3]. If the supersymmetry is also broken, then the components of this superfield will become massive, except for the Nambu-Goldstone boson. The only question is: how is the scale of supersymmetry breaking related to the scale of spontaneous breaking of the continuous global symmetry? Naively, we would expect them to be the same. On the other hand, it is desirable in the implementation of the axionic solution [4, 5] of the strong CP problem [6] that the $U(1)_{PQ}$ symmetry breaking scale, presumably of order 10^9 to 10^{12} GeV, be much greater than the supersymmetry breaking scale, presumably of order 1 TeV.

In this paper, this problem is revisited in the case of the most general superpotential of 3 singlet superfields. The relationship of the 2 arbitrary mass scales of this superpotential to the $U(1)$ breaking scale is clarified, particularly with respect to a simplified form [7] which is widely used in the literature. The possibility of having a "seesaw" mass spectrum with new bosons and fermions at the TeV scale is pointed out.

2 Superpotential of 3 Singlet Superfields

Consider 3 singlet superfields $\hat{\phi}_1$, $\hat{\phi}_2$, and $\hat{\chi}$, transforming as $+1$, -1, and 0 under a global $U(1)$ symmetry. The following simple superpotential,

$$\hat{W} = f\hat{\chi}(\hat{\phi}_1\hat{\phi}_2 - \Lambda),\tag{1}$$

is invariant under $U(1)$ and is widely used in axionic supersymmetric models. The idea is that the supersymmetric minimum of the corresponding scalar potential is given by

$$v_1 v_2 = \Lambda, \tag{2}$$

where $v_{1,2} = \langle \phi_{1,2} \rangle$, which breaks $U(1)$ spontaneously and gives rise to the axion. However, it is clear that Eq. (1) is missing the allowed term $\mu_{12} \hat{\phi}_1 \hat{\phi}_2$ which must be set equal to zero by hand to obtain Eq. (2). Nevertheless, as long as the supersymmetry is exact, the condition $\mu_{12} = 0$ is maintained to all orders in perturbation theory. In reality, we know the supersymmetry is broken, hence a natural lower bound for μ_{12} is $M_{SUSY} \sim 1$ TeV. This has the important phenomenological consequence that all particles associated with the axion (including the axino) must not be much lighter than M_{SUSY}.

Instead of the simplified form of Eq, (1), consider the most general superpotential of $\hat{\phi}_1$, $\hat{\phi}_2$, and $\hat{\chi}$, i.e.

$$\hat{W} = \frac{1}{2} m \hat{\chi}^2 + \frac{1}{3} h \hat{\chi}^3 + \mu \hat{\phi}_1 \hat{\phi}_2 + f \hat{\chi} \hat{\phi}_1 \hat{\phi}_2. \tag{3}$$

Under the transformation $\hat{\chi} \to \hat{\chi} + u$, we have

$$\hat{W} \to \frac{1}{2} m u^2 + \frac{1}{3} h u^3 + u(m + hu)\hat{\chi} + \frac{1}{2}(m + 2hu)\hat{\chi}^2 + \frac{1}{3} h \hat{\chi}^3 + (\mu + fu)\hat{\phi}_1 \hat{\phi}_2 + f \hat{\chi} \hat{\phi}_1 \hat{\phi}_2. \tag{4}$$

If we now require

$$\mu + fu = 0, \quad f\Lambda = -u(m + hu), \tag{5}$$

then after dropping the constant term, we have

$$\hat{W} = \frac{1}{2}(m + 2hu)\hat{\chi}^2 + \frac{1}{3} h \hat{\chi}^3 + f \hat{\chi}(\hat{\phi}_1 \hat{\phi}_2 - \Lambda), \tag{6}$$

which becomes Eq. (1) in the limit $m \to 0$, $h \to 0$, but Λ remains finite. Thus the <u>hidden assumptions</u> of Eq. (1) are that m is very small, but u (hence μ) is very large, such that Λ is the order of their product. Since m should be bounded from below by M_{SUSY}

and μ from above by M_{Planck}, a reasonable value for Λ is indeed 10^3 GeV $\times$ 10^{19} GeV $=$ $(10^{11}$ GeV$)^2$ and suitable for the axion scale.

Consider now Eq. (6) with $m \neq 0$ and $h \neq 0$. The two fundamental mass scales m and μ of Eq. (3) may be chosen arbitrarily and independently as long as each is greater than M_{SUSY}. However, the choice $m \sim M_{Planck}$ and $\mu \sim M_{SUSY}$, i.e. opposite to that of Eq. (1), is very natural and has interesting consequences, to be shown below.

3 Breaking of the U(1) Symmetry

Let the superfields $\hat{\phi}_1$ and $\hat{\phi}_2$ be redefined as $v_1 + \hat{\phi}_1$ and $v_2 + \hat{\phi}_2$, with $v_1 v_2 = \Lambda$, then

$$\hat{W} = \frac{1}{2}(m + 2hu)\hat{\chi}^2 + \frac{1}{3}h\hat{\chi}^3 + f\hat{\chi}(v_2\hat{\phi}_1 + v_1\hat{\phi}_2 + \hat{\phi}_1\hat{\phi}_2), \tag{7}$$

which shows that the superfields

$$\hat{\chi}, \quad \hat{\eta} \equiv \frac{v_2\hat{\phi}_1 + v_1\hat{\phi}_2}{\sqrt{v_1^2 + v_2^2}}, \tag{8}$$

are massive with a mass matrix given by

$$\mathcal{M} = \begin{bmatrix} m + 2hu & fv \\ fv & 0 \end{bmatrix}, \tag{9}$$

where $v = \sqrt{v_1^2 + v_2^2}$ and that the superfield

$$\hat{\zeta} \equiv \frac{v_1\hat{\phi}_1 - v_2\hat{\phi}_2}{\sqrt{v_1^2 + v_2^2}} \tag{10}$$

is massless. In terms of $\hat{\chi}$, $\hat{\eta}$, and $\hat{\zeta}$,

$$\hat{W} = \frac{1}{2}(m + 2hu)\hat{\chi}^2 + \frac{1}{3}h\hat{\chi}^3 + fv\hat{\chi}\hat{\eta} + scf\hat{\chi}(\hat{\eta}\hat{\eta} - \hat{\zeta}\hat{\zeta}) + (c^2 - s^2)f\hat{\chi}\hat{\eta}\hat{\zeta}, \tag{11}$$

where $s = \sin\theta$, $c = \cos\theta$, and $\tan\theta = v_2/v_1$.

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In anticipation of the assumed symmetric soft breaking of the supersymmetry which will set $v_1 = v_2$, we simplify Eq. (11) by taking $s = c = 1/\sqrt{2}$. In addition, using $v^2 = 2\Lambda$ and Eq. (5), we assume $|\mu| << |m|$. Hence

$$fv = f\sqrt{2\Lambda} \simeq \sqrt{2\mu m} << |m| \tag{12}$$

in Eq. (9). Thus the heavy superfield $\hat{\chi}$ can be integrated out and the effective $\hat{\eta}$ acquires a "seesaw" mass, i.e.

$$m_\eta \simeq -\frac{f^2 v^2}{m} \simeq -2\mu. \tag{13}$$

The effective superpotential of $\hat{\eta}$ and $\hat{\zeta}$ is then given by

$$\hat{W} = \frac{1}{2}m_\eta\hat{\eta}\hat{\eta} - \frac{m_\eta}{2v}\hat{\eta}(\hat{\eta}\hat{\eta} - \hat{\zeta}\hat{\zeta}). \tag{14}$$

The corresponding contribution to the Lagrangian is

$$
\begin{aligned}
-\mathcal{L}_{int} = & \ |m_\eta\eta - (3m_\eta/2v)\eta^2 + (m_\eta/2v)\zeta^2|^2 + |(m_\eta/v)\eta\zeta|^2 \\
& + \ [(m_\eta/2)\tilde{\eta}\tilde{\eta} - (3m_\eta/2v)\eta\tilde{\eta}\tilde{\eta} + (m_\eta/2v)\eta\tilde{\zeta}\tilde{\zeta} + (m_\eta/v)\zeta\tilde{\zeta}\tilde{\eta} + h.c.]
\end{aligned} \tag{15}
$$

Since the scalar field η and the fermion field $\tilde{\eta}$ are massive, they can be integrated out to obtain the effective Lagrangian of ζ and $\tilde{\zeta}$. To lowest order, it should be of the form

$$-\mathcal{L}_{eff} = a|\zeta|^2 + b|\zeta|^4 + c\tilde{\zeta}\tilde{\zeta}. \tag{16}$$

At tree level, it is clear that $a = c = 0$, and b is given by the diagrams of Fig. 1, i.e.

$$b = \left(\frac{m_\eta}{2v}\right)^2 - \frac{(m_\eta^2/2v)^2}{m_\eta^2} = 0, \tag{17}$$

which is exactly as expected. In fact, $a = b = c = 0$ is true to all orders in perturbation theory as long as the supersymmetry is not broken because $\hat{\zeta}$ is a massless superfield and the symmetry of Eq. (14) forbids the $\hat{\zeta}^3$ term in $\hat{W}_{eff}$.

4   Breaking of the Supersymmetry

To break the supersymmetry softly, we go back to Eq. (3) and consider

$$V_{soft} = \mu_\chi^2 |\chi|^2 + \mu_0^2(|\phi_1|^2 + \phi_2|^2) + [B\mu\phi_1\phi_2 + Af\chi\phi_1\phi_2 + h.c.] \tag{18}$$

The minimum of the complete scalar potential is then given by

$$
\begin{aligned}
V_{min} &= (mu + hu^2 + fv_1v_2)^2 + (\mu + fu)^2(v_1^2 + v_2^2) \\
&+ \mu_\chi^2 u^2 + \mu_0^2(v_1^2 + v_2^2) + 2B\mu v_1 v_2 + 2Afuv_1v_2.
\end{aligned}
\tag{19}
$$

Using $v_1 = v_2 = v/\sqrt{2}$, and $|\mu| << |m|$, we find

$$v^2 \simeq -\frac{2mu}{f}, \tag{20}$$

with u given by

$$3f^2u^2 + 2f(2\mu + A)u + \mu^2 + \mu_0^2 + B\mu = 0. \tag{21}$$

In the limit $V_{soft} = 0$, we recover the supersymmetric solution of Eq. (5) as expected. Now if we set $\mu \sim M_{SUSY}$ as well as μ_0, B, and A, then $u \sim M_{SUSY}$ is assured.

We can again use the seesaw approximation to integrate out χ and obtain the masses of η and ζ. Instead of $m_\eta \simeq -2\mu$ and $m_\zeta = 0$ in the supersymmetric limit, we obtain

$$m_{Re\eta}^2 \simeq 2\mu[2\mu - 4(\mu + fu) + 2A], \tag{22}$$

$$m_{Im\eta}^2 \simeq 2\mu[2\mu + 2A] + 2(\mu + fu)^2 + 2\mu_0^2, \tag{23}$$

$$m_{Re\zeta}^2 = 2(\mu + fu)^2 + 2\mu_0^2, \tag{24}$$

$$m_{Im\zeta}^2 = 0. \tag{25}$$

Since $\mu_{12}\tilde{\phi}_1\tilde{\phi}_2$ is an allowed soft supersymmetry breaking term, the mass of the fermion $\tilde{\eta}$ is now $|-2\mu + \mu_{12}|$ and that of $\tilde{\zeta}$ is $|\mu_{12}|$.

5   Physical Consequences

Starting with Eq. (3), we have shown that there are 2 interesting limits. If $m \to 0$ and $\mu \to \infty$ with $v \sim \sqrt{m\mu}$ finite, we get Eq. (1) with the following particle spectrum. The superfields $\hat{\chi}$ and $\hat{\eta}$ are heavy with mass fv. The scalar field ζ contains the axion as its phase, with v as the scale of $U(1)$ symmetry breaking, but the mass of $|\zeta|$ is the order of M_{SUSY} as is the mass of the axino $\tilde{\zeta}$. At the other extreme with $m \sim M_{Planck}$ and $\mu \sim M_{SUSY}$, we get also $v \sim \sqrt{m\mu}$ which is now fixed at $\sim 10^{11}$ GeV, and suitable for the axion scale. The particle spectrum in this case is different. The superfield $\hat{\chi}$ is heavy with mass m, but both η and $|\zeta|$, as well as $\tilde{\eta}$ and $\tilde{\zeta}$ have masses the order of M_{SUSY}.

6   Concluding Remarks

In general, the values of m and μ are arbitrary and independent, and protected against large radiative corrections by the supersymmetry. So as long as they are greater than M_{SUSY}, the field theory is well-behaved and the $U(1)$ breaking scale of $\sqrt{m\mu}$ is generated. If only one mass scale is allowed by the choice of $U(1)$ charge assignments as in a recent axionic supersymmetric model for neutrino masses [8], then this mechanism is not available, resulting in the breaking of $U(1)$ at that one mass scale [9]. Nevertheless, $M_{SUSY} << v$ is possible in all these examples.

Acknowledgement

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[9] Using 4 singlet superfields with PQ charge assignments +2, 0, –1, –2, we can again have 2 mass scales m and μ. The axion scale of $\sqrt{m\mu}$ is then generated and can be identified with the mass scale of the heavy singlet neutrino as in Ref.[8].
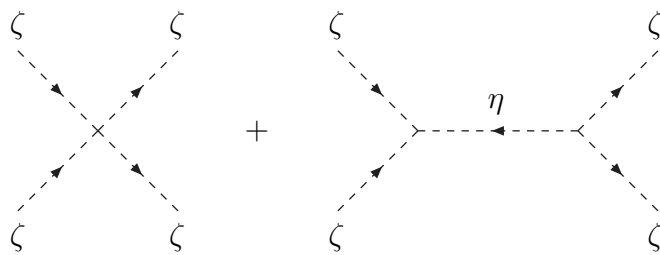
Figure 1: Contributions to the effective quartic coupling $|\zeta|^4$.